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VALIDATING GPS BASED MEASUREMENTS FOR VEHICLE CONTROL

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ABSTRACT

The increasingly widespread use of the Global Positioning System (GPS) in determining the location of vehicles raises the possibility of using the information provided by GPS for vehicle control purposes. The use of a multi-antenna GPS system provides the ability to measure not only position and velocity, but vehicle heading and sideslip as well. This paper presents a validation of a GPS based system with an automotive grade two-axis optical sensor. The results show excellent agreement between the two sensor systems, confirming the accuracy of the GPS based system even in highly dynamic situations. Although any GPS based system is subject to dropouts from driving under trees and bridges, cornering stiffness estimates obtained when GPS is available enable construction of a vehicle state observer for use in the absence of GPS.

NOMENCLATURE

a	Distance from vehicle CG to front axle
a_y	Lateral acceleration
α_f	Front tire slip angle
α_r	Rear tire slip angle
b	Distance from vehicle CG to rear axle
β	Sideslip angle
$C_{\alpha f}$	Front cornering stiffness
$C_{\alpha r}$	Rear cornering stiffness
d	Longitudinal position
δ	Steering angle at road wheels
F_{brake}	Braking force
F_{yf}	Front lateral tire force
F_{yr}	Rear lateral tire force
I_z	Vehicle moment of inertia
m	Vehicle mass
r	Yaw rate
R	Tire radius
s	Longitudinal slip
U_{abs}	Absolute velocity over ground
U_{tire}	Tire velocity
U_x	Longitudinal velocity
U_y	Lateral velocity
ω	Tire angular velocity

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INTRODUCTION

As stability and control systems advance, the need for accurate measurements of the dynamics of the vehicle becomes increasingly important. The automotive industry has long relied on optical sensors for high accuracy measurements of the dynamics of vehicles on test tracks, but the high cost of such sensors makes them impossible to integrate into production vehicles. Consequently, current stability and control systems cannot make use of the accurate measurements that optical sensors are able to provide. In particular, current stability and control systems must function without accurate knowledge of the vehicle's velocity and sideslip angle.

The increasingly widespread use of the Global Positioning System (GPS) in determining the location of vehicles raises the possibility of using the information provided by GPS for vehicle control purposes. A GPS based navigation unit for help with driving directions is already standard on many passenger vehicles. The use of a multi-antenna GPS system provides the ability to measure not only position and velocity, but vehicle heading and sideslip as well [1] [2]. While the update rate of GPS measurements is typically slow compared to the dynamics of a vehicle, the system can be augmented with a set of accelerometers and gyros to provide real-time vehicle state measurements [3] [4]. A combination of GPS antennas, accelerometers and gyros could provide measurements at the level of optical sensors, but at a price more feasible for automotive production.

Of course, any GPS based measurement system has its limitations. GPS signals cannot travel through trees, bridges, or buildings. In addition, the quality of the GPS solution is dependent upon the geometry of the satellites in view [5]. However, during the time the vehicle does have access to GPS measurements, it is possible to estimate vehicle parameters that can then be used to construct a model of the system. This model can be used along with other sensor measurements during a GPS outage to construct an observer to estimate the vehicle states.

This work seeks to validate GPS based measurements of vehicle velocity and sideslip with an automotive grade two-axis optical sensor. Experimental results show excellent agreement between the two sensor systems, confirming the accuracy of the GPS based system even in highly dynamic situations. Preliminary results suggest that a vehicle state observer could provide information about sideslip even when GPS based measurements are not available.

THE NEED FOR ACCURATE VELOCITY AND SIDESLIP

There are numerous examples of how accurate knowledge of the vehicle's velocity and sideslip angle could be used to improve vehicle stability and control systems. Two such examples are included here.

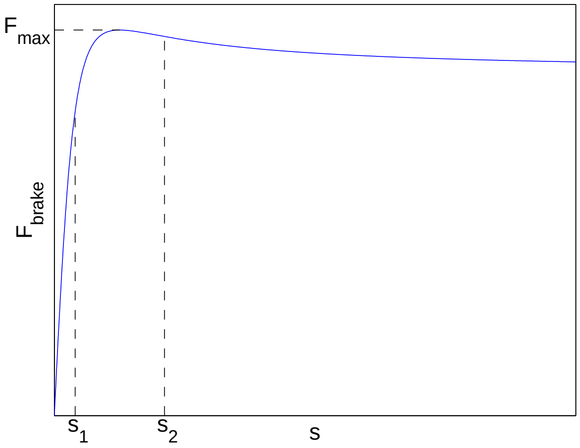

Figure 1. TYPICAL TIRE CHARACTERISTIC, BRAKING FORCE AS A FUNCTION OF SLIP ANGLE

Slip Control Systems: Improved Slip Control

A tire generates a traction force depending on the longitudinal slip, s, of the tires, which is the relative difference between the absolute velocity over ground of the vehicle, U_{abs}, and the velocity of the tire, U_{tire}:

$$s = \frac{U_{abs} - U_{tire}}{U_{abs}} \tag{1}$$

where:

$$U_{tire} = R\omega \tag{2}$$

where R is the tire radius and ω is the angular velocity of the wheel. Today's anti-lock braking systems (ABS) primarily use the information from the individual wheel speed sensors to determine the actual angular velocity of the wheels. From these measurements, the ABS controller estimates the angular acceleration of each wheel and the absolute velocity over ground, from which longitudinal slip estimates can be derived. The ABS controller then uses these estimates to decide whether to further increase, hold or decrease the individual wheel pressures [6]. Figure 1 shows the braking force profile of a typical tire. In order to minimize the stopping distance in an emergency braking situation, today's ABS controllers cycle around the peak force, operating between s_1 and s_2. The average braking force in this region is less than the maximum braking force, F_{max}. The problem here is that the longitudinal wheel slip is calculated from an inaccurate estimate of the vehicle speed over ground. Without knowledge of the absolute vehicle speed over ground, U_{abs}, the ABS controller is unable to maximize the braking force, and this in turn limits the performance of the control system.

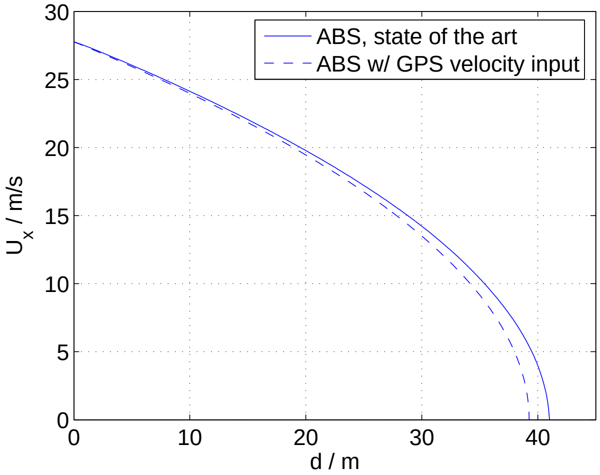

Figure 2. COMPARISON OF STOPPING DISTANCE WITH AND WITHOUT KNOWLEDGE OF VEHICLE'S ABSOLUTE VELOCITY

With direct measurement of speed over ground from GPS, the performance of ABS systems could be improved. Figure 2 illustrates the magnitude of improvement that could theoretically be achieved if the ABS system could maintain the peak braking force, F_{max}. The solid line represents the stopping distance of a car with longitudinal slip variation characteristic of a conventional vehicle. The dashed line represents the case where accurate estimation of longitudinal slip enables the controller to maintain the maximum value of braking force. The difference in stopping distance is significant, in this case on the order of 5 to 10 percent of the total distance. Thus, knowledge of the absolute velocity over ground would consequently help to make vehicles safer since their braking performance could be notably improved.

Stability Control: Improved Instability Detection

Today's stability control systems like active yaw control (Dynamic Stability Control (DSC), Electronic Stability Program (ESP), Active Steering, etc.) typically use lateral acceleration and yaw rate as inputs to determine the lateral state of the vehicle. However, to determine the lateral slip of the vehicle and detect instability, sideslip angle information is needed [7].

Figure 3 shows that the sideslip angle information is much more useful in determining a critical situation than yaw rate (or lateral acceleration). In the depicted maneuver, the vehicle performs a 1.6 rad step steer at about 10 m/s on ice with the tires reaching the limit of adhesion. It is obvious that the sideslip angle substantially deviates from the expected value that is derived from a reference model. About 1 second after the vehicle enters the unstable state, the sideslip deviates from the expected value by about 0.06 rad, while the yaw rate only deviates from the expected value by about 0.12 rad/s. After 2 seconds, the sideslip deviation is about 0.16 rad and the yaw rate deviation has decreased to about 0.06 rad/s. Taking into account that control systems typically use a threshold to avoid inappropriate control interventions, a yaw based stability control system would have difficulties in the denoted maneuver. In contrast, a sideslip based control system would detect the instability. This is the conclusion drawn in [7], which also concludes that a multiple input controller, sensing yaw rate and sideslip angle, would be most effective. Such a controller would help prevent unstable vehicle states without inappropriate interventions, thus providing effective assistance in maneuvering a vehicle through critical situations. However, such a multiple input controller cannot be realized without accurate knowledge of the sideslip angle.

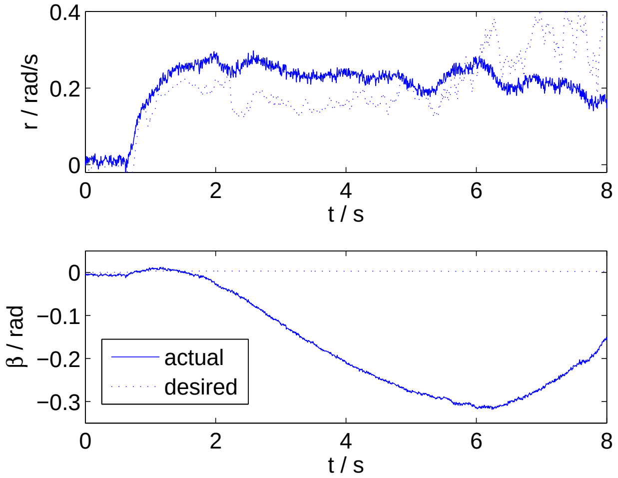

Figure 3. LOW FRICTION MANEUVER SHOWING LARGE SIDESLIP AND SMALL YAW RATE

EXPERIMENTAL SETUP AND VERIFICATION

In order to validate GPS based measurements of vehicle velocity and sideslip with an optical sensor, a vehicle was equipped with both measurement systems. The vehicle's Control Area Network (CAN) output was used as a third sensor system. After the initial high friction testing confirming the validity of the GPS based system, additional tests were performed with just the GPS based system in place in order to illustrate the applicability of the system over a wide range of driving conditions.

GPS Based System

The use of a multi-antenna GPS system provides the ability to measure not only position and velocity, but vehicle heading and sideslip as well [1] [4]. A BeeLine two-antenna system from NovAtel is used in conjunction with a NovAtel single-antenna system. Figure 4 shows the GPS antennas mounted on the roof of the vehicle. The single antenna, mounted along the center line

Figure 4. GPS ANTENNAS AND OPTICAL SENSOR MOUNTED ON EXPERIMENTAL VEHICLE

of the vehicle, provides velocity information. The two antennas for the BeeLine receiver, mounted on either side of the center antenna, provide heading and sideslip information. This system is not dependent on the driving conditions as long as GPS is available.

Since the update rate of GPS measurements is typically slow compared to the dynamics of a vehicle, the system is augmented with a set of automotive grade accelerometers and gyros to provide real-time vehicle state measurements [3]. Three accelerometers and three gyros are mounted in pairs along perpendicular axes in a small cube unit to serve as an Inertial Navigation System (INS) unit for the purpose of augmenting the GPS measurements. Figure 5 shows the INS unit mounted on a bar rigidly attached to the frame of the vehicle. The motion of the bar relative to the vehicle was found to be negligible.

A MATLAB/Simulink/XPC environment was used to collect and process the data from the GPS receivers and the INS unit. Figure 5 also shows the "XPC box" used to collect the data.

Optical Sensor

The automotive industry has long relied on optical sensors for high accuracy measurements of the dynamics of vehicles on test tracks. One such industry standard is a Datron V Sensor from Corrsys-Datron. This automotive grade two-axis sensor provides the ability to measure both longitudinal and lateral velocities, and thus sideslip as well. The sensor works by correlating pictures of the ground along two perpendicular axes, as shown in Fig. 6. The velocity of the vehicle along these two axes is computed and then transformed by the sensor to the longitudinal and lateral velocities at the location of the sensor. With knowledge of the yaw rate of the vehicle and the location of the sensor relative to the center of mass of the vehicle, the vehicle's longitudinal and lateral velocities at the vehicle's center of mass can then be computed. This information can then be used to directly compute the sideslip angle. Optical sensors such as this one do not work in all driving conditions and cannot, for instance, be used over puddles of water or in snow storms. Gaps in the pavement may also result in inaccurate measurements.

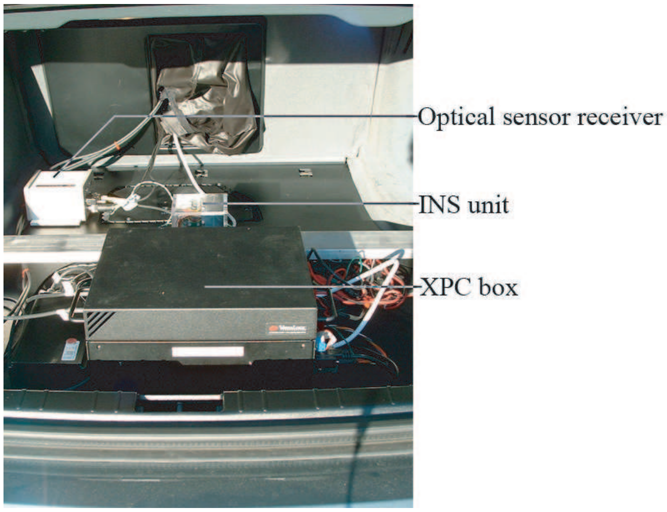

Figure 5. INSIDE TRUNK OF EXPERIMENTAL VEHICLE

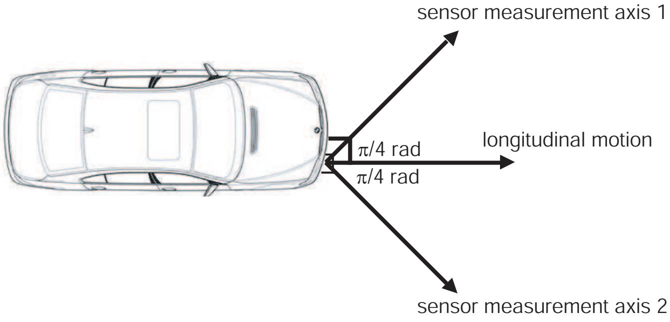

Figure 6. THE OPTICAL SENSOR MEASURES ALONG TWO PERPENDICULAR AXES

Figure 4 shows the Datron V Sensor mounted to the front bumper of the vehicle and Fig. 5 shows the sensor's processing unit mounted inside the trunk. The optical sensor was mounted such that the two measurement axes were at $\pi/4$ rad with respect to the longitudinal motion of the vehicle, as depicted in Fig. 6. The signal from the Datron V Sensor was read into the XPC box via a serial link to allow a direct comparison between the measurements from both systems without requiring that different time vectors be matched up in post processing.

Control Area Network

In order to measure the steering wheel angle, the vehicle's Control Area Network (CAN) was also connected to the XPC box. This also provided information on the wheel speeds and the vehicle speed over ground estimate calculated by the ABS controller.

Verification with the Optical Sensor and High Friction Testing

A variety of maneuvers were conducted with summer tires on dry concrete with both the GPS/INS system and optical sensor operational. The maneuvers included straight running at constant speed, hard acceleration/deceleration, steady state cornering, quasi-steady state cornering with fixed steering wheel angle, step steers, single/double lane changes, parking, and sinusoidal steering (fixed frequency/increasing amplitude and fixed amplitude/increasing frequency). These maneuvers were chosen in order to span a wide range of velocities and accelerations, and to include both slow and quick changes in vehicle dynamics. The optical sensor was assumed to be the truth reference in all of the high friction testing.

Velocity. Figure 7 shows the speed of the vehicle as measured by the GPS/INS system and the optical sensor during an emergency braking maneuver from an initial speed of about 33 m/s. The steering wheel was held constant at 0 rad during the maneuver. The most important thing to note is that the results show excellent agreement between the GPS/INS system and the optical sensor, confirming the accuracy of the GPS/INS system. The next important thing to note is that the signal from the GPS/INS system is significantly smoother than the signal from the optical sensor. Since the GPS/INS system does not rely on properties of the road surface, it is not susceptible to disturbances such as pavement irregularities that the optical sensor incorporates into its measurements. As a comparison to what is currently available in a production vehicle, the vehicle's CAN measurement of speed is also included in Fig. 7. The ABS system engages shortly after the driver presses on the brake pedal. Since the CAN speed is derived from wheel speed measurements, it includes the cycling of the ABS system in its measurement of vehicle speed, and consequently does not give very accurate information.

Sideslip. Figure 8 shows the sideslip of the vehicle as measured by the GPS/INS system and the optical sensor during a lane change maneuver at about 30 m/s at the limits of tire adhesion. Figure 9 shows the yaw rate of the vehicle during this maneuver. As in the velocity case, there is excellent agreement between the two systems in steady state behavior and good agreement in transient behavior, again confirming the accuracy of the GPS/INS system.

Time Delay. In both Figs. 7 and 8, it appears that the signal from the GPS/INS system leads the signal from the optical sensor. The information from the optical sensor seems to have almost a 100 ms time delay with respect to the information from the GPS/INS system. This may well have been a result of the specific sensor processing used and is not necessarily representative of optical sensor performance. The point of this comparison is not that the GPS/INS system is inherently faster; these plots serve mainly to demonstrate that the GPS/INS system introduces minimal delay in the measurement.

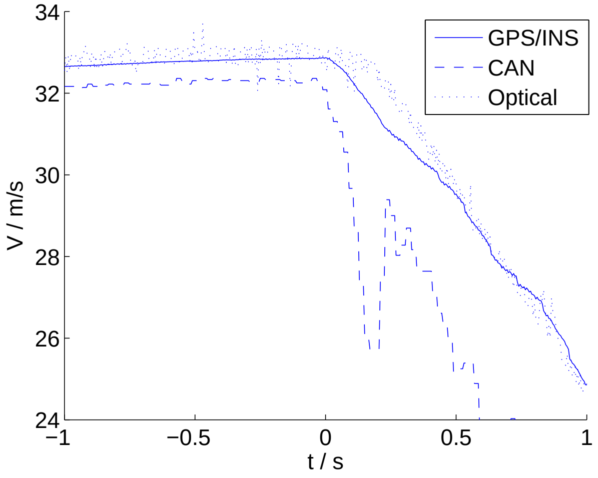

Figure 7. SPEED OF VEHICLE MEASURED BY THREE SENSOR SYSTEMS

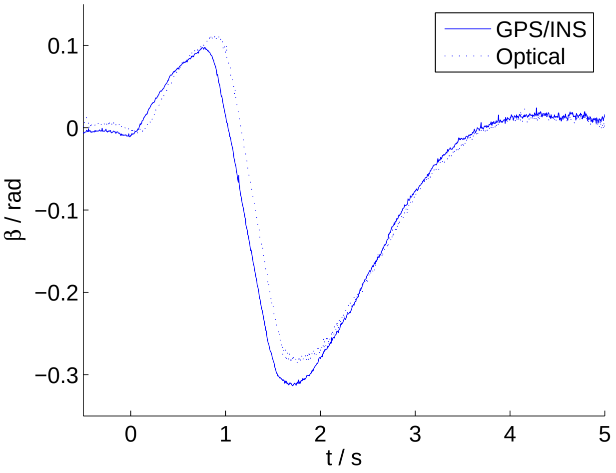

Figure 8. SIDESLIP OF VEHICLE AS MEASURED BY GPS/INS SYSTEM AND OPTICAL SENSOR

Low Friction Testing

Testing was also performed on snow and ice with winter tires in order to illustrate the ability of the GPS/INS system to detect

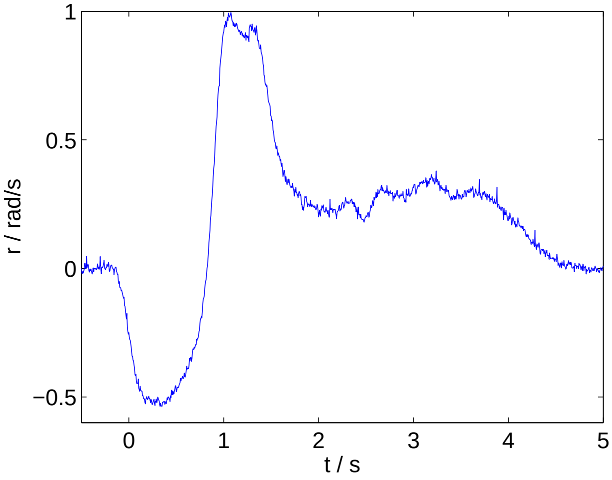

Figure 9. YAW RATE OF VEHICLE DURING MANEUVER SHOWN IN FIG. 8

large sideslip angles. The maneuvers performed were the same as in the high friction case. There was no notable difference in the system's performance as compared to the high friction tests. The vehicle's dynamics were changed dramatically, generating larger sideslip angles more frequently, but the GPS/INS system appeared to sense these motions accurately. Due to the pavement type (snow and ice), the optical sensor was not used during these tests. However, the measurements that the GPS/INS system makes are independent of the road surface, and therefore the different road surface was not expected to have any impact on the performance of the system.

Figure 3 shows one example of a low friction maneuver (1.6 rad step steer at about 10 m/s on ice) during which the driver did not counteract and the vehicle slid with a large sideslip angle. The system appears to track the vehicle's motion accurately, and even under these conditions there are no signal drift effects apparent since the signal returns to its 0 rad value when the vehicle comes to a stop.

Different Tires

In order to check the sensitivity of the GPS/INS system, the test vehicle was equipped with both summer and winter tires to change the vehicle characteristics. Sinusoidal steering maneuvers, such as the one depicted in Fig. 10, were performed on dry pavement. Figures 11 and 12 show that the system generates data accurate enough to identify tire characteristics. The ellipses represent the hysteresis resulting from the tire dynamics, and the lines represent the best linear fit to the data. Approximate front and rear cornering stiffnesses are 138,000 N/rad and 267,000 N/rad, respectively, for the summer tires. For the winter tires, the front and rear cornering stiffnesses are approximately 124,000 N/rad and 188,000 N/rad, respectively. As expected, the summer tires have higher cornering stiffness than the winter tires. In addition, Fig. 11 shows a noticeable difference between the front and rear summer tires, while Fig. 12 shows less of a difference between the front and rear winter tires. This is also reasonable, since the weight distribution is almost equal between the two axles, and the front and rear summer tires have different dimensions whereas the winter tires do not.

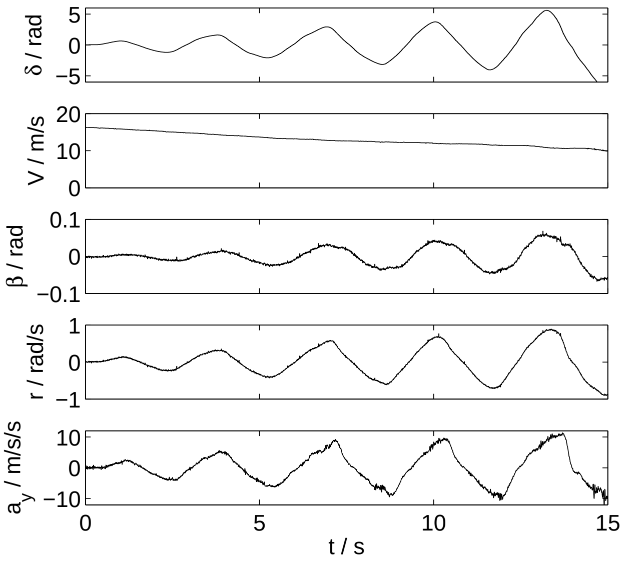

Figure 10. SINUSOIDAL STEERING MANEUVER

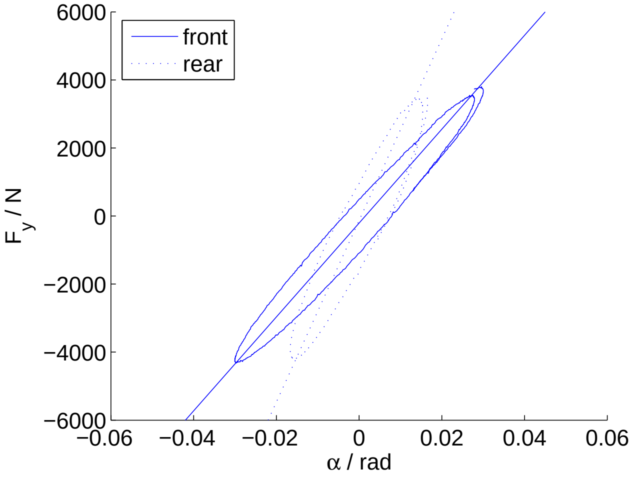

Figure 11. TIRE CHARACTERISTICS FOR SUMMER TIRES ON DRY PAVEMENT

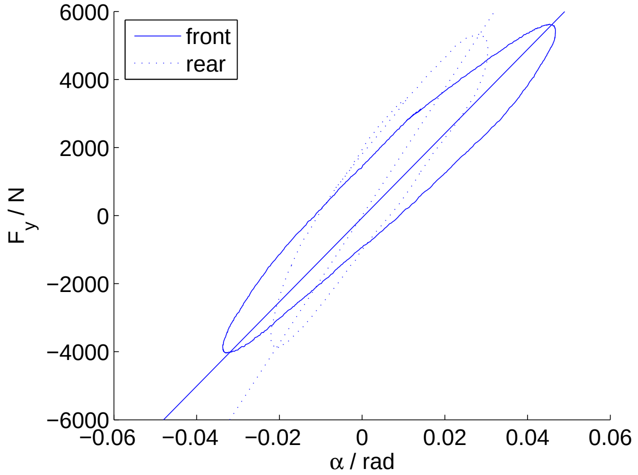

Figure 12. TIRE CHARACTERISTICS FOR WINTER TIRES ON DRY PAVEMENT

GPS OUTAGES AND DROPOUTS

The comparison between the GPS/INS system and the automotive grade two-axis optical sensor suggests that GPS based measurements are accurate enough to be used for vehicle control purposes. However, GPS is not available during all driving situations. The GPS signal is blocked by trees, bridges, and tall buildings, so driving in such an environment would prevent any GPS based measurements from being made. One solution to this problem would be to only use the control systems that rely on GPS based measurements when GPS is available, and to turn them off when GPS is unavailable. This is not desirable, since control systems that cannot function all of the time cannot be as effective as those that can function continuously. Therefore, there needs to be a way to obtain vehicle state information in the event of a GPS outage.

There are numerous examples of observers that have been developed for estimating lateral velocity and sideslip angle that do not depend on GPS. For example, in [8], measurements of lateral acceleration and yaw rate are used to estimate the lateral velocity. In [9], these same two measurements are combined with estimates of the tire lateral forces to estimate the lateral velocity. Lateral velocity information is then combined with longitudinal velocity information to form an estimate of sideslip. Other examples can be found in [10].

During the time that GPS is available, it is possible to collect information about the system and construct a model. For example, cornering stiffness can be estimated as shown in Figs. 11 and 12. The model can then be used along with other sensor measurements to estimate the vehicle states when GPS measurements are not available. In contrast to many other methods, which use lateral acceleration measurements to estimate lateral velocity (and then combine the estimate with longitudinal velocity to estimate sideslip angle), the method presented here treats the lateral acceleration as a scalar function of the vehicle states and inputs. This formulation allows for the direct estimation of sideslip angle from lateral acceleration measurements.

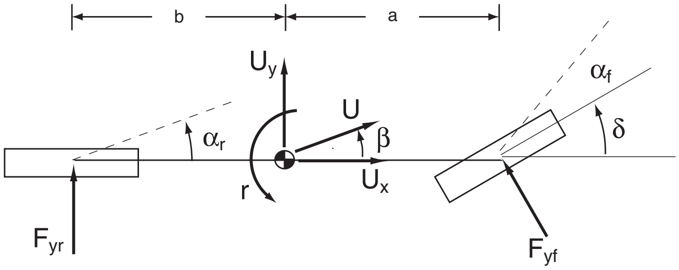

Figure 13. PLANAR VEHICLE MODEL

Vehicle Model

The vehicle is modeled with the linear "bicycle model," Fig. 13, in which the vehicle is treated as a mass moving in the plane. Assuming small angles, the equations of motion for the vehicle can be written as:

$$\dot{x} = Ax + B\delta \tag{3}$$

where δ is the steering angle at the road wheels and:

$$x = \begin{bmatrix} \beta \\ r \end{bmatrix} \quad A = \begin{bmatrix} -\frac{c_o}{mU_x} & -(\frac{c_1}{mU_x{}^2}+1) \\ -\frac{c_1}{I_z} & -\frac{c_2}{I_zU_x} \end{bmatrix} \quad B = \begin{bmatrix} \frac{C_{\alpha f}}{mU_x} \\ \frac{aC_{\alpha f}}{I_z} \end{bmatrix} \tag{4}$$

where β is the sideslip angle, r is the yaw rate, m is the mass of the vehicle, U_x is the longitudinal velocity, I_z is the moment of inertia, a is the distance from the vehicle CG to the front axle, $C_{\alpha f}$ is the front axle cornering stiffness, and:

$$c_o = C_{\alpha f} + C_{\alpha r} \tag{5}$$

$$c_1 = aC_{\alpha f} - bC_{\alpha r} \tag{6}$$

$$c_2 = a^2C_{\alpha f} + b^2C_{\alpha r} \tag{7}$$

where $C_{\alpha r}$ is the rear axle cornering stiffness and b is the distance from the vehicle CG to the rear axle.

When GPS is not available, the sideslip angle, β, cannot be measured. However, the INS unit or the vehicle's CAN system can be used to measure the yaw rate, r, and the lateral acceleration, a_y. Assuming knowledge of the longitudinal velocity, the relation:

$$a_y = U_x(\dot{\beta} + r) \tag{8}$$

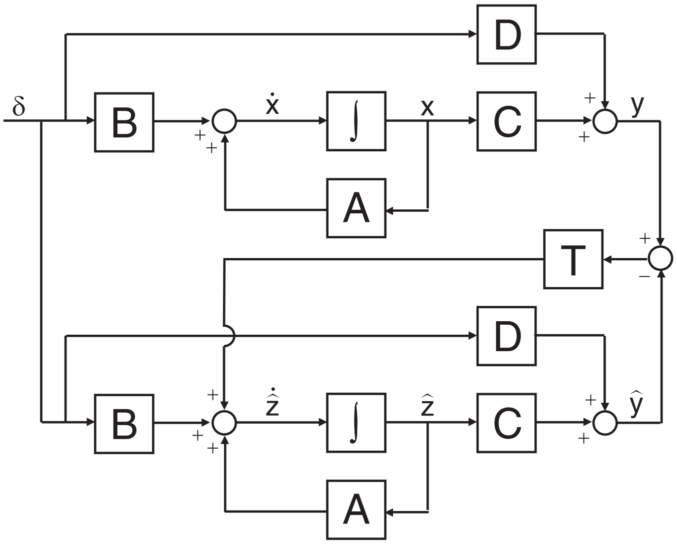

Figure 14. OBSERVER STRUCTURE

allows the measurements from the system, y, to be expressed as a linear combination of the vehicle states, x, and the input, δ:

$$y = Cx + D\delta \tag{9}$$

where:

$$y = \begin{bmatrix} a_y \\ r \end{bmatrix} \quad C = \begin{bmatrix} -\frac{c_o}{m} & -\frac{c_1}{mU_x} \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} \frac{C_{\alpha f}}{m} \\ 0 \end{bmatrix} \tag{10}$$

In general, the sideslip angle, β, in the system described by Eqn. (3) is unobservable when the yaw rate, r, is the only measurement and the vehicle approaches neutral steering. By adding the measurement of lateral acceleration, a_y, the sideslip angle becomes observable.

Observer Design

An observer to estimate sideslip from the available measurements of yaw rate and lateral acceleration can now be constructed. To do this, a simple linear observer is used in the form of Fig. 14 where:

$$\dot{\hat{z}} = A\hat{z} + B\delta + T(y - \hat{y}) \tag{11}$$

$$\hat{y} = C\hat{z} + D\delta \tag{12}$$

where $\hat{z}$ is the estimate of the unknown states, x, and $\hat{y}$ is the estimate of the measurement vector, y. The observer feedback matrix, T, is computed such that the error dynamics, $\dot{\hat{z}} - \dot{x}$, are stable. Equation 11 can be written more compactly as:

$$\dot{\hat{z}} = A_{obs}\hat{z} + B_{obs}u \tag{13}$$

where:

$$A_{obs} = [A - TC] \quad B_{obs} = \begin{bmatrix} B - TD & T \end{bmatrix} \quad u = \begin{bmatrix} \delta \\ y \end{bmatrix} \tag{14}$$

The standard bicycle model assumes a constant velocity in the longitudinal direction, which is not a valid assumption in general. Hence, the observer feedback matrix, T, is computed for an approximate speed, and the matrices A_{obs} and B_{obs} are computed at each time step with the current velocity information. This velocity information would normally come from GPS, which is assumed to be unavailable. However, there are other sources of approximate velocity information. For instance, we could use information from the vehicle's CAN system. While this information is not as accurate as the velocity information received from GPS, it is generally close enough to use for the purpose of generating the observer matrices, A_{obs} and B_{obs}.

Preliminary Results

This technique was applied to some of the data sets acquired during the testing maneuvers. Since the two-antenna system was operational during these tests, we can compare the observer estimate to the values measured during the experiments. Figure 15 shows a step steer maneuver on which this technique was applied. The steering input, δ, and the lateral acceleration, a_y, were taken from the vehicle's CAN system. The yaw rate, r, was taken from the measurements made by the INS unit.

Figures 16 and 17 show the results of applying this observer. The estimated value of the yaw rate tracks the actual yaw rate exactly, which was expected since the yaw rate was a state we could measure. The estimated value of the sideslip does not exactly track the measured sideslip. However, the difference between the observer estimate and the measurement from the two-antenna system is not very large. The measurements of yaw rate, r, and lateral acceleration, a_y, both contain a small overshoot at around 3 sec. Consequently, the observer estimate for sideslip, $\hat{\beta}$, contains an overshoot not present in the original data. Despite this, the general trend is maintained.

CONCLUSIONS AND FUTURE WORK

The comparison between the GPS/INS system and the automotive grade two-axis optical sensor confirmed that the GPS/INS system does provide accurate measurements of velocity and sideslip even in highly dynamic situations. The results also show that the GPS based system is more robust to disturbances such

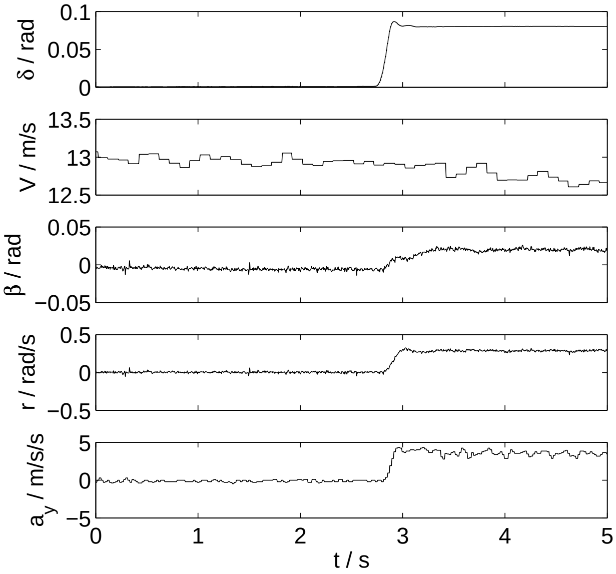

Figure 15. STEP STEER MANEUVER

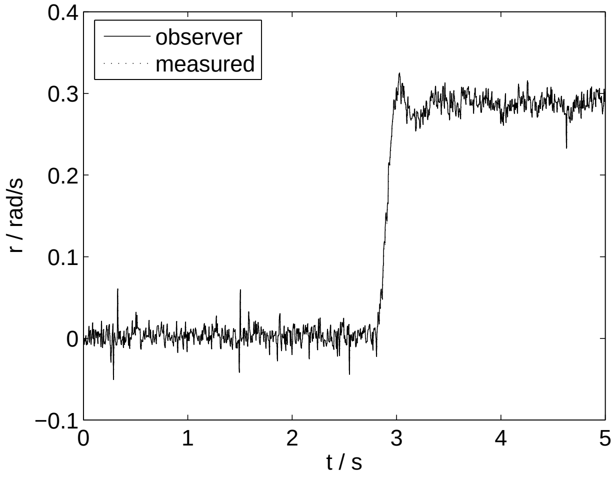

Figure 16. OBSERVER ESTIMATE OF YAW RATE

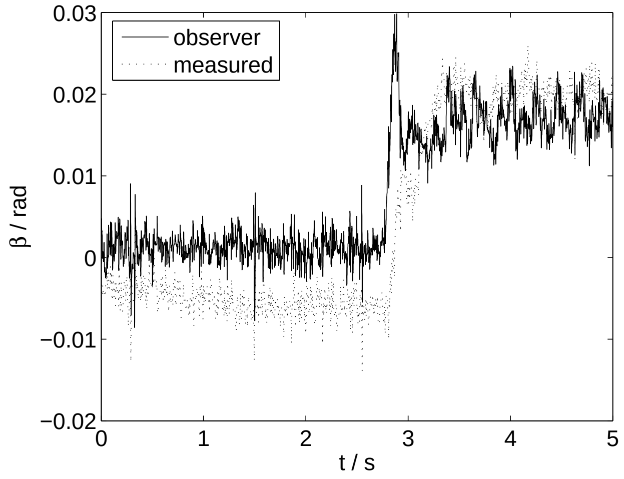

Figure 17. OBSERVER ESTIMATE OF SIDESLIP

as pavement irregularities that the optical-based sensor suffers from, suggesting that the GPS based system would also be more amenable for implementation into a stability or control system.

The vehicle state observer described here can be used to augment a GPS based measurement system to give sideslip information even in the event of a GPS outage. Expressing the lateral acceleration as a linear combination of the vehicle states and inputs allows for the direct estimation of the sideslip angle. Although the estimate of sideslip does not exactly match the measurement from the GPS/INS system, the estimate does follow the same trends as the measurement. This observer was developed using the linear bicycle model, but could easily be developed using a nonlinear formulation that would include nonlinear tire dynamics. This type of formulation would most likely improve the observer's ability to estimate sideslip.

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